On Relations Between Electroweak Hierarchy Problem, Fluctuating Three Branes And General Covariance

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Abstract

We inquire whether a resolution to the electroweak hierarchy problem could reside in symmetries that relate the bosonic Weinberg-Salam Lagrangian with a higher dimensional generally covariant theory. For this we consider a three-brane that moves under the influence of a seven dimensional pure Hilbert Einstein-like generally covariant theory. We introduce a change of variables that combines the conformal scale of the metric tensor with the brane fluctuations, so that the conformal scale becomes the modulus and the fluctuations become the angular field degrees of freedom of a polarly decomposed Higgs. When we assume that the four dimensional space-time background of the generally covariant theory is locally conformally flat and that the internal space is a squashed three-sphere, we arrive at one massless and three massive vector fields akin those in the Weinberg-Salam model and recover all the familiar ingredients of the symmetry broken bosonic Weinberg-Salam model, except that there is no bare Higgs mass. The Higgs mass is subject to dimensional censorship, its presence is forbidden by general covariance. This proposes that a resolution to the electroweak hierarchy problem might well reside in higher dimensions, in a Ward-Takahashi like identities for general covariance that relate a non-vanishing Higgs mass to dynamical breaking of general covariance. Moreover, the two electroweak gauge couplings are both determined by the squashing parameter of the internal three sphere and when we impose the condition that the vector boson masses must be in line with custodial symmetry we arrive at the classical level to the Weinberg angle $\sin^2\theta_W \approx 0.296$.

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I. INTRODUCTION

Experiments at LHC should soon expose the Higgs particle and reveal the mechanism of the electroweak symmetry breaking. Eventually LHC should also allow us to scrutinize the fine details of the Higgs sector and give some hints how Physics behaves at even higher energy scales. A *Big Issue* is the inherent instability of the mass of the standard electroweak Higgs. In the Weinberg-Salam model the natural value of the Higgs mass is proportional to the cut-off scale of the theory [1], [2], thus a delicate fine-tuning becomes necessary in order for it to attain realistic values [3]. As a consequence the predominant point of view is that the Weinberg-Salam model is an effective low energy theory, valid only to a scale no higher than a few TeV. Several approaches are pursued to resolve this electroweak hierarchy problem. These include supersymmetric theories [4], little Higgs models [5], extra dimensions [6] and many others.

In the present article we shall explore (perhaps) new ways to address the electroweak hierarchy problem. We search for a resolution from such symmetries of the Weinberg-Salam Lagrangian that have a higher dimensional origin and interpretation. Our modus operandi is the Kaluza-Klein approach [7], [8]. But instead of attempting to formulate the electroweak theory in a higher dimensional space-time as in a conventional Kaluza-Klein approach, we restrict our attention to symmetry structures in the four dimensional electroweak theory that allow for a higher dimensional Kaluza-Klein interpretation: The conventional Kaluza-Klein approach leads to still unresolved problems in particular in the fermionic sector [9], and for this reason we prefer not to start from any *a priori* prescribed higher dimensional theory. In our view, the question whether we live in a higher dimensional space-time or not remains a rather interpretational and maybe even philosophical one. A resolution may well be in a hybrid picture where bosonic variables fluctuate into higher dimensions while fermions are constrained into four, if indeed a higher dimensional interpretation that goes beyond pure symmetry considerations turns out to have some real advantages.

Our point of view is rather pragmatic and of diagnostic nature: We inquire what is a minimal amount of higher dimensional structure present in the Weinberg-Salam model that we need to look into, in order to properly address and hopefully resolve the electroweak hierarchy problem in the limited context of the four dimensional bosonic Weinberg-Salam Lagrangian. For this we consider a mathematical construct, that combines a four-dimensional space-time manifold $\mathbb{M}_4$ with some hypothetical higher dimensional internal manifold. It appears likely that whatever the structure of the internal manifold, it should somehow relate to the squashed three-sphere since on a squashed three-sphere the commutators of Killing vectors coincide with the Lie algebra of $SU(2) \times U(1)$ [7]. Consequently we think it is reasonable to assume that *any* approach to electroweak interactions that involves higher dimensions, somehow relates to the structure of a squashed three sphere, and thus we take it to be our internal manifold.

In our higher dimensional construct, we then bring in a generally covariant theory with an action similar to that of pure Einstein gravity (even though we make no claims that the theory has anything to do with gravity - our focus is on general covariance and the "gravity" is simply a mathematical construct to realize it). We inspect its symmetry structure in a background that is determined by a suitably chosen metric tensor that we subject to a Kaluza-Klein dimensional reduction onto massless modes. Since we are only interested in how symmetries with a higher dimensional interpretation act on the field content of the Weinberg-Salam model, we can safely ignore any issues related to complications that may arise due to dilaton fields or from an infinite tower of massive modes. Questions on higher dimensional stability of the construction are also too technical and complex to be addressed here, for the present purposes it suffices to note that experimental observations show no sign of instability in the standard electroweak theory.

From the conventional point of view of a Kaluza-Klein compactification our approach can be interpreted so that we are inquiring how to *minimally embed* the Weinberg-Salam Lagrangian into a higher dimensional context, rather than *deriving*

it from a consistent Kaluza-Klein truncation and with no additional fields. We find it quite unlikely that a full and consistent truncation with no added four dimensional fields is even possible, at least we are not familiar with any. Instead of addressing this issue which is at the hearth of any fully consistent higher dimensional approach, we take a minimalistic point of view to simply query that whatever the higher dimensional theory (if it indeed exists, also for fermions) it must lead to certain symmetry structures that we wish to expose and apply to address the electroweak hierarchy problem. In this manner we arrive at a four dimensional $SU(2) \times U(1)$ gauge theory, which is essentially the standard electroweak theory: We recover the correct mass assignments for the vector fields, but there is *no* Higgs mass and the two gauge couplings are not independent but related and determined by the parameter describing the squashing of the three-sphere. The reason why the Higgs mass is absent is rather telling: A bare Higgs mass term breaks general covariance. Consequently, *if* there is a good reason to insist on general covariance as it appears here, there should be some Ward-Takahashi type identity that ensures that also in the quantized electroweak theory there is no Higgs mass term, thus no hierarchy problem. However, there is a non-vanishing Higgs condensate that has a dynamical origin in a A^2-condensation of the intermediate vector bosons [10]-[12].

It is notable that even though our construction leaves no room even for a primordial Higgs field, we do obtain the correct masses for the $SU(2) \times U(1)$ gauge fields. However, the mechanism that equips the three intermediate bosons with their mass differs from the standard approach based on spontaneous breaking of $SU_L(2) \times U_Y(1)$ into electromagnetic $U(1)$ by the Higgs field: We introduce a three-brane that in our mathematical construct asymptotically coincides with the physical four dimensional space-time, but can locally fluctuate into the higher dimensional manifold where it moves under the influence of the higher dimensional Hilbert- Einstein like generally covariant interaction. The brane fluctuations become "eaten up" by the longitudinal modes of three vector bosons and as a consequence they become massive, their masses being determined by the brane tension in combination of the squashing parameter.

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We show that the brane fluctuations are precisely the angular components of the Standard Model Higgs field, in a polar decomposition of the Higgs. The remaining field degree of freedom, the Higgs modulus, resides in the conformal scale of the four dimensional Hilbert-Einstein like generally covariant Lagrangian. If we specify to a conformally flat space-time we obtain by a change of variables a Lagrangian that is very much like the original Weinberg-Salam model in the conformally invariant (Coleman-Weinberg) limit of the Higgs potential, in the *flat* space-time $\mathbb{R}^4$ and with the correct mass relations for the intermediate vector bosons. Except that we now have also a Ward-Takahashi like condition that the underlying general covariance should remain unbroken, preventing the presence of a bare Higgs mass, also when radiative corrections are accounted for.

Furthermore, from the value $\alpha = 1/137$ of the fine structure constant we estimate at the classical level and in the purely bosonic theory the value

$$\sin^2 \theta_W \approx 0.296$$

for the Weinberg angle. This is an experimental constraint for our higher dimensional interpretation, and even though we derive it in the purely bosonic theory and at the classical level, the result is surprisingly close to the experimentally measured value [13], [14]. Indeed, the final theory is almost *verbatim* equal to the Weinberg-Salam model, except that the couplings now have a common origin and there is no bare Higgs mass.

In the next section we describe our approach in the context of the Abelian Higgs Model, in the wider context of full Kaluza-Klein reduction. Here there are no technical issues with the dilaton, and for completeness we display the entire dilaton sector. We show how the massive $U(1)$ gauge boson and the neutral scalar that together determine the particle content of the Abelian Higgs model in its spontaneously broken phase, emerge from the massless modes of a five dimensional Kaluza-Klein compactification *with no primordial Higgs field*. However, the conventional Higgs field with the ensuing vector boson mass can be fully reconstructed by introducing a three brane

that locally fluctuates into the fifth dimension with dynamics governed by the Nambu action. The brane fluctuations are described by a variable that corresponds to the phase of the Higgs field, and its modulus emerges from the conformal scale of the metric tensor when we consider the theory in a locally conformally flat space-time and introduce certain changes of variables. After reconstructing the Higgs we arrive at the standard classical Abelian Higgs model in the space-time $\mathbb{R}^4$, with a Coleman-Weinberg type potential for the Higgs field at the classical level. In the quantum theory there is a Higgs condensate it is determined by the A^2-condensation of the gauge field. But since a bare Higgs mass breaks general covariance, a massive Higgs field remains forbidden.

Our full-fledged Kaluza-Klein analysis that includes the dilaton sector reveals that while the dilatons have an essential rôle in stabilizing the theory, the presence of dilatons is less important when we are only interested in relating the symmetry structure of the higher dimensional theory with that of its Kaluza-Klein descendant. The dilaton has no effect on the emergence of vector mass in the dimensionally reduced theory, and in particular it has no effect on the Higgs mass that is forbidden by general covariance. This gives us confidence that when we proceed to the non-Abelian case where we only consider the symmetry structure, there is no reason to explicitly consider the effect of dilatons.

In Section 3 we describe the Weinberg-Salam model for the present purposes. In particular, we introduce a generalization of the changes of variables that in Section 2 enabled us to relate the Abelian Higgs model to a Kaluza-Klein reduction of five dimensional generally covariant theory with Hilbert-Einstein action. In Section 4 we consider certain mathematical properties of the squashed three sphere. We derive a number of relations that will be useful for us when we proceed to construct the Weinberg-Salam model. In Section 5 we first show how to introduce the $SU(2) \times U(1)$ gauge structure of the Weinberg-Salam model, by employing a Kaluza-Klein reduction that starts from a seven dimensional Hilbert-Einstein action with an internal space that is a squashed three sphere. Following the Abelian example we explain

how the vector fields acquire their masses when we introduce a three brane that is asymptotically stretched into the non-compact directions, but is locally allowed to fluctuate into the squashed three sphere. We first consider a Nambu action for the three brane. This yields us a version of the Weinberg-Salam model with a wrong relation between the intermediate boson masses, and a local $SU_L(2) \times SU_R(2)$ custodial symmetry which is explicitly broken. We then modify the Nambu action by introducing a parameter that measures the deviation of the mass matrix from the point of custodial symmetry. When we choose this parameter to correspond to the value where custodial symmetry is recovered we obtain the standard Weinberg-Salam model in the Coleman-Weinberg limit of the Higgs potential but *again* with the additional Ward-Takahashi like condition that a bare Higgs mass is forbidden by general covariance, and the couplings corresponding to the gauge groups $SU(2)$ and $U(1)$ are determined by a single parameter that describes the squashing of the three sphere. Furthermore, we show that from the familiar low energy value of the fine structure constant $\alpha = 1/137$ we find a value for the Weinberg angle that is quite close to the observed value. We also argue that as in the Abelian case there is a Higgs condensate that has its origin in the A^2-condensation of the intermediate vector bosons.

Finally, in order to avoid the familiar tricky issues that are associated with the conformal scale and its analytic continuation [15] we work exclusively in a space-time with Euclidean signature. However, we see no problems in extending our results to a space-time with Minkowskian metric: Since there is no reason why the generally covariant Hilbert-Einstein action that we introduce describes gravity, we may as well choose the overall sign of the Minkowskian scalar curvature to be opposite to that in Einstein gravity.

II. ABELIAN MASS FROM FLUCTUATING THREE BRANE

Here we show how the standard Abelian Higgs model with its Higgs effect can be derived from the brane world, but without a primordial Higgs field. We start from

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the familiar, full Kaluza-Klein decomposition of the five dimensional metric tensor [7]

$$ds^2 = g_{ij}dx^i dx^j = e^{2\alpha\phi}g_{\mu\nu}dx^\mu dx^\nu + e^{2\beta\phi}g_{55}\left(d\theta + A_\mu dx^\mu\right)^2 \tag{1}$$

We label $i, j, ... = 0, ..., 4$ and $\mu, \nu, ... = 0, ..., 3$, and α and β are parameters. The compact coordinate $\theta \in [0, 2\pi)$ describes a circle $\mathbb{S}^1$ with a local radius that depends on the scalar field ϕ and the constant g_{55} has dimensions of length squared. Under a x^μ dependent reparametrization of $\mathbb{S}^1$

$$\theta \to \theta + \varepsilon(x^\mu) \tag{2}$$

$$A_\mu \to A_\mu - \partial_\mu \varepsilon$$

the metric (1) remains intact. For A_μ this is the familiar $U(1)$ gauge transformation.

We are interested in the (low energy) limit where the fields become independent of the compact coordinate θ. In this limit we take the metric components $g_{\mu\nu}$ and the fields ϕ, A_μ to depend only on the four dimensional coordinates x^μ. The ensuing five dimensional generally covariant (Hilbert-Einstein) action with cosmological constant $\Lambda_{(5)}$

$$S = \frac{1}{16\pi G_{(5)}} \int d^5x \sqrt{g_{(5)}} \left\{R - 2\Lambda_{(5)}\right\}$$

becomes

$$= \frac{1}{16\pi G} \int d^4x \sqrt{g} \left[e^{(\beta+2\alpha)\phi} \{R + 6\alpha(\alpha+\beta)\nabla_\mu\phi\nabla^\mu\phi\} \right.$$

$$\left. + \frac{1}{4}e^{3\beta\phi}g_{55}F_{\mu\nu}F^{\mu\nu} - 2\Lambda e^{(\beta+4\alpha)\phi} \right] \tag{3}$$

We have here removed a boundary term from partial integration, R is the Ricci scalar and g is the determinant of the four dimensional $g_{\mu\nu}$, and G is akin the four dimensional Newton's constant.

In the conventional approach [7] one now proceeds by assuming that the parameters α and β obey the following relation

$$\beta = -2\alpha \tag{4}$$

But here we proceed instead with the complementary choice

$$\beta \neq -2\alpha$$

This allows us to introduce the change of variables

$$d\phi = \frac{1}{\beta + 2\alpha} \frac{d\sigma}{\sigma}$$

which obviously can not be introduced if (4) is assumed. Next we define

$$\kappa = \frac{\beta}{\beta + 2\alpha}$$

and implement the conformal transformation

$$g_{\mu\nu} \rightarrow \sigma g_{\mu\nu}$$

followed by the additional change of variables,

$$d\chi = \sqrt{\frac{3}{2}} \kappa \frac{d\sigma}{\sigma}$$

In this way we find that (3) becomes

$$= \frac{1}{16\pi G} \int d^4x \sqrt{g} \left[R - \nabla_\mu \chi \nabla^\mu \chi + \frac{1}{4} e^{\sqrt{6}\chi} g_{55} F_{\mu\nu} F^{\mu\nu} - 2\Lambda e^{-\sqrt{\frac{2}{3}}\chi} \right] \tag{5}$$

independently of α and β, provided of course that $\beta \neq -2\alpha$. This action has the same functional form as the four dimensional Brans-Dicke action in interaction with Maxwellian electrodynamics, with a coupling that depends on the dilaton field χ. The Liouville-like instability of the dilaton ground state is apparent.

It is notable that since the dilaton has no charge the vector field remains massless. In order to obtain a massive vector field and a relation to the Abelian Higgs model in its spontaneously broken phase we proceed to construct a gauge invariant mass term for the U(1) gauge field in (5). For this we consider a three-brane that stretches along the non-compact directions of the five dimensional space-time. This brane is locally described by a scalar function

$$\theta = h(x_\mu) \tag{6}$$

The induced metric tensor on the brane is obtained by pulling-back the five-metric (1) with the help of the basis vectors on the brane,

$$E_\mu{}^i = \delta_\mu{}^i + \frac{\partial h}{\partial x^\mu} \delta_5{}^i \tag{7}$$

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This gives the induced brane metric

$$G^{ind}_{\mu\nu} = E_\mu{}^i E_\nu{}^j g_{ij} = e^{2\alpha\phi}\left(g_{\mu\nu} + g_{55}e^{2(\beta-\alpha)\phi}(A_\mu + \partial_\mu h)(A_\nu + \partial_\nu h)\right) \qquad (8)$$

Note that this metric is *invariant* under the $U(1)$ isometry (3): A local shift in the brane position

$$h(x) \rightarrow h(x) + \varepsilon(x)$$

can be compensated by the shift

$$A_\mu \rightarrow A_\mu - \partial_\mu \varepsilon(x)$$

in the gauge field.

We assume that the dynamics of the three-brane is governed by the Nambu action

$$S_{brane} = T \int d^4x \sqrt{G^{ind}} \qquad (9)$$

where T is a dimensionfull parameter, the brane tension. We compute the determinant of the metric,

$$G^{ind} = \det[G^{ind}_{\mu\nu}] = e^{8\alpha\phi} \cdot \det[g_{\mu\nu}] \cdot \left(1 + g_{55}e^{2(\beta-\alpha)\phi}(A_\mu + \partial_\mu h)g^{\mu\nu}(A_\nu + \partial_\nu h)\right) \quad (10)$$

In the limit of small brane fluctuations $\partial_\mu h$ we then get from (9), (10) the following (low energy) brane action,

$$S_{brane} = T \int d^4x \sqrt{g}\, e^{4\alpha\phi} \left(1 + \frac{1}{2}e^{2(\beta-\alpha)\phi}g_{55}(A_\mu + \partial_\mu h)g^{\mu\nu}(A_\nu + \partial_\nu h) + ...\right) \quad (11)$$

Here the combination

$$\mathcal{J}_\mu = A_\mu + \partial_\mu h \qquad (12)$$

is manifestly invariant under the reparametrizations (3). When we implement in (9), (11) the changes of variables that took us from (3) to (5) we get the combined action

$$S + S_{brane} = \int d^4x \sqrt{g}\left[\frac{1}{16\pi G}\{R - \nabla_\mu\chi\nabla^\mu\chi\}\right.$$

$$\left. + \frac{1}{4}\frac{g_{55}}{16\pi G}e^{\sqrt{6}\chi}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}Tg_{55}e^{-\sqrt{\frac{2}{3}}\chi}\mathcal{J}_\mu\mathcal{J}^\mu + Te^{-\sqrt{\frac{8}{3}}\chi} - \frac{2\Lambda}{16\pi G}e^{-\sqrt{\frac{2}{3}}\chi}\right] \qquad (13)$$

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Here

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{J}_\nu - \partial_\nu \mathcal{J}_\mu - [\partial_\mu, \partial_\nu] h \tag{14}$$

and its last term vanishes whenever the brane fluctuations $h(x)$ are twice continuously differentiable. It is also notable that all α and β dependence has again disappeared.

Let us consider the dilaton potential term in (13),

$$V(\chi) = T e^{-\sqrt{\frac{8}{3}}\chi} - \frac{2\Lambda}{16\pi G} e^{-\sqrt{\frac{2}{3}}\chi}$$

We observe that the presence of the brane has introduced an additional term that allows us to stabilize the dilaton, we now have a nontrivial local minimum at

$$\chi_{min} = -\sqrt{\frac{3}{2}} \ln \left[\frac{\Lambda}{16\pi GT} \right]$$

Consequently if we redefine

$$\mathcal{J}_\mu \rightarrow e\mathcal{J}_\mu \tag{15}$$

where

$$e = \frac{1}{16\pi G} \sqrt{\frac{\Lambda^3}{g_{55}T^3}}$$

and define

$$m^2 = \frac{g_{55}\Lambda}{16\pi G}$$

and redefine

$$\frac{\Lambda^2}{16\pi GT} \rightarrow \Lambda$$

then at the local minimum of the dilaton potential the kinetic term for $\mathcal{J}_\mu$ acquires its correct canonical normalization and the action (13) becomes

$$S(\chi_{min}) = \int d^4x \sqrt{g} \left[\frac{1}{16\pi G}(R - 2\Lambda) + \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{m^2}{2}\mathcal{J}_\mu \mathcal{J}^\mu \right] \tag{16}$$

Notice in particular that we can simultaneously have a large "Planck's mass", a small "cosmological constant", and a gauge vector mass that is independent of the other two, thus at this level we avoid delicate fine tuning problems. There is also no primordial Higgs field even though the vector boson has acquired a mass, and even though the mass appears from dilaton interaction term when the dilaton field acquires

its classical ground state value the mechanism is different from the Higgs mechanism. In particular the local ground state of the dilaton potential is not degenerate.

We now show that in a locally conformally flat space-time with metric

$$g_{\mu\nu} = \frac{\rho^2}{\kappa^2}\delta_{\mu\nu} \tag{17}$$

the action (16) *exactly* coincides with that of the Abelian Higgs Model in $\mathbb{R}^4$, in its spontaneously broken phase. We have here chosen ρ to have the dimension of mass, and we have introduced an *a priori* arbitrary mass parameter κ to ensure that the components of the metric tensor $g_{\mu\nu}$ are dimensionless.

We start from the 3+1 dimensional Abelian Higgs multiplet, a complex scalar field φ and a $U(1)$ gauge field A_μ. We introduce a change of variables to another set of six independent fields J_μ, ρ and θ

$$\begin{aligned}\varphi &\leftrightarrow \rho \cdot e^{i\theta} \\ A_\mu &\leftrightarrow J_\mu = \tfrac{i}{2e\rho^2}\left[\varphi^*(\partial_\mu - ieA_\mu)\varphi - c.c.\right]\end{aligned} \tag{18}$$

This change of variables is invertible whenever $\rho \neq 0$, the Jacobian is ρ. When we introduce a $U(1)$ gauge transformation that acts on φ and A_μ in the usual way, the fields J_μ and ρ are $U(1)$ gauge invariant; the vector J_μ is known as the *supercurrent* in applications to superconductivity. In terms of these variables the familiar $U(1)$ gauge invariant classical action of the Abelian Higgs model

$$S = \int d^4x \left\{ \frac{1}{4}F_{\mu\nu} + |(\partial_\mu - ieA_\mu)\varphi|^2 + \lambda|\phi|^4 \right\} \tag{19}$$

becomes

$$S = \int d^4x \left\{ \frac{1}{4}(J_{\mu\nu} - \sigma_{\mu\nu})^2 + (\partial_\mu\rho)^2 + e^2\rho^2 J_\mu^2 + \lambda\rho^4 \right\} \tag{20}$$

where

$$J_{\mu\nu} = \partial_\mu J_\nu - \partial_\nu J_\mu$$

and the distribution

$$\sigma_{\mu\nu} = \frac{1}{e}[\partial_\mu, \partial_\nu]\theta \tag{21}$$

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is the string tensor that describes vorticity, in line with the second term in (14). Its support in $\mathbb{R}^4$ coincides with the world-sheets of vortex cores. Except for θ in (21) there are no gauge dependent variables present in the action (20). Furthermore, if gauge transformations entail only at least twice continuosly differentiable functions, (21) is gauge invariant. Thus in the absence of (singular) vortex configurations we have the remarkable result that the $U(1)$ gauge dependence of the Abelian Higgs Model can be entirely removed by a mere change of variables. For this, there is *no* need to introduce any fixing of gauge nor any kind of symmetry breaking mechanism by the Higgs field [16]-[19]. We shall see that this persists in the case of the Weinberg-Salam Model.

As in [18] we identify the variable ρ in (20) with the conformal scale of a metric tensor like (17). With this metric tensor we can then write [18] the classical action (20) in the following manifestly generally covariant form,

$$ S \ = \ \int d^4x \sqrt{g} \left\{ \frac{1}{16\pi G}(R + 2\Lambda) + \frac{1}{4}g^{\mu\nu}g^{\lambda\eta}J_{\mu\lambda}J_{\nu\eta} + e^2\kappa^2 g^{\mu\nu}J_\mu J_\nu \right\} \qquad (22) $$

We have here introduced

$$ G = \frac{3}{8\pi\kappa^2} \qquad (23) $$

and

$$ \Lambda = 3\kappa^2\lambda \qquad (24) $$

and for simplicity we include the string tensor (21) in the definition of $J_{\mu\nu}$. With the present identifications (16) and (22) clearly coincide, as we asserted. Furthermore, remarkably the vector field in the Abelian Higgs model has acquired a mass $\sqrt{2}e\kappa$ even though no explicit symmetry breaking and in particular no Higgs effect has taken place. But the theory now resides in an emergent space-time that is *different* from the $\mathbb{R}^4$ where the original Lagrangian (19) endures. This emergent space-time dissolves away when ρ vanishes. In particular, on the world-sheet of an Abrikosov type vortex where (21) is nontrivial we must have $\rho = 0$, otherwise the energy diverges. The metric (17) then vanishes and the curvature scalar

$$ R = -6\left(\frac{\kappa^2}{\rho}\right)^2 \cdot \frac{\Box\rho}{\rho} $$

has an integrable singularity. As a consequence the Abrikosov vortices of the Abelian Higgs model can be viewed as space-time singularities in the emergent, locally conformally flat space-time.

We emphasize that we have not included any bare Higgs mass term in (19). If we include a bare Higgs mass in (19) it spoils the manifestly covariant form of (22) and we loose the relation between the generally covariant (16) and (19). While at the level of (19), (22) the general covariance can be viewed as a pure coincidence, at the level of (16) general covariance is a symmetry of the theory and adding a term corresponding to a bare Higgs mass in(19) explicitely break general covariance. If we accept a point of view that there is something deeper between (16) and (19) than pure coincidence, there can not be any bare Higgs mass in (19) either. Consequently, if the theory (19) is regularized and quantized in a manner that continues to relate it to (16) and thus respects the generally covariant interpretation (22) *no* bare Higgs mass term can emerge as a quantum correction, Higgs mass is forbidden by an appropriate Ward-Takahashi identity that reflects the underlying generally covariant interpretation, and the relation between (19) and (16). We may say that the Higgs mass is subject to *dimensional censorship.*

In the sequel we shall show that the present construction persists in the non-Abelian context of Weinberg-Salam model, thus such a dimensional censorship might be a natural resolution to the electroweak hierarchy problem: A mass term for the Higgs field is not consistent with the symmetries of the theory as these symmetries have their origin in higher dimensional general covariance and a bare Higgs mass breaks this general covariance.

However, this does not prevent ρ from having a non-trivial ground state value: Even when the perturbative contributions that are not consistent with the interpretation in terms of general covariance are removed, it has been argued in [10]-[12] both on general grounds and using numerical lattice simulations that in a quantum gauge theory the condensate

$$< J_\mu^2 > = \ \pm\Delta^2 \tag{25}$$

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is non-vanishing. Here we have added the sign to reflect the fact that in *Minkowski* space the condensate can be either time-like or space-like: Following [20] we expect that there is a phase transition with order parameter

$$< J_4^2 > - < J_i^2 > = \pm \Delta^2$$

and according to (20) the sign corresponds to positive *resp.* negative Higgs mass

$$e^2 J_\mu^2 \rho^2 \rightarrow e^2 < J_\mu^2 > \rho^2 \sim \pm e^2 \Delta^2 \rho^2 = \pm e^2 \Delta^2 |\phi|^2$$

that is the phase transition is between the symmetric and broken Higgs phases in the conventional parlance. Indeed, it has been proposed that the quantity (25) determines a natural and *gauge invariant* [10]-[12] dimension-two condensate in a gauge theory. From (20) we then estimate that in the London limit where $\rho = \rho_0$ is a constant corresponding to the conventional situation where the Higgs field is in a translationally invariant ground state, we have the non-vanishing condensate value

$$\rho_0^2 = \frac{e^2 \Delta^2}{\lambda} \tag{26}$$

Finally we comment on the following: As such it should not come as a surprise that a Poincare invariant field theory can be written in a generally covariant form. For this all one needs is to implement a transformation from the Cartesian to a generic coordinate system, the result always has a generally covariant form. However, the *peculiarity* in the present case is that now the metric tensor is constructed from one of the field variables, and that the Hilbert-Einstein action makes an appearance. In fact, it has been proposed that any unitary four dimensional field theory that possesses both Poincare and rigid scale symmetry is invariant under the entire conformal group $SO(5, 1)$ [21]. The results of [22], [23] in the case of (special) conformally invariant $\lambda \phi^4$ and of [12], [18], [19] in the case of (special) conformally invariant Yang-Mills-Higgs theories then suggest that the $SO(5, 1)$ special conformal symmetry in $\mathbb{R}^4$ can be extended to include invariance under local conformal transformations, and the ensuing theory can be cast in a manifestly generally covariant form with the conformal scale constructed from the field variables [24].

III. SUPERCURRENTS AND THE WEINBERG-SALAM MODEL

We now proceed to the non-Abelian $SU_L(2) \times U_Y(1)$ invariant Weinberg-Salam model, where our goal is to relate its symmetry structure with that of a higher dimensional gravity theory to address the electroweak hierarchy problem. The present Section describes how the pertinent Lagrangian (22) is derived, and the remaining Sections are devoted to relate the ensuing (accidental?) general covariance to a higher dimensional local symmetry.

The bosonic part of the Weinberg-Salam Lagrangian is

$$\mathcal{L}_{\mathrm{WS}} = \frac{1}{4}\vec{G}_{\mu\nu}^2(A) + \frac{1}{4}F_{\mu\nu}^2(Y) + |D_\mu\Phi|^2 + \lambda|\Phi|^4 \tag{27}$$

We work in a flat spacetime with Euclidean signature, and follow the notation of [26]: The matrix-valued $SU_L(2)$ isospin gauge field is

$$\widehat{A}_\mu \equiv A_\mu^a \tau^a = \vec{A}_\mu \cdot \vec{\tau}$$

with τ^a the isospin Pauli matrices, and Y_μ is the Abelian $U_Y(1)$ hypergauge field. The field strengths are

$$\vec{G}_{\mu\nu}(A) = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu - q\,\vec{A}_\mu \times \vec{A}_\nu\,, \tag{28}$$

$$F_{\mu\nu}(Y) = \partial_\mu Y_\nu - \partial_\nu Y_\mu\,. \tag{29}$$

and the $SU_L(2) \times U_Y(1)$ covariant derivative is

$$D_\mu = \mathbb{1}\,\partial_\mu - i\frac{q}{2}\widehat{A}_\mu - i\frac{q'}{2}Y_\mu\,\mathbb{1}\,, \tag{30}$$

where $\mathbb{1}$ is the 2×2 unit matrix in the isospin space.

Notice that as in the case of Abelian Higgs model, we do *not* add any (bare) mass term to the complex isospinor Higgs boson Φ, the Higgs potential is conformally invariant, of the Coleman-Weinberg form. It turns out that as in the Abelian case there is no need for a conventional kind of a Higgs effect. Instead, the gauge boson

masses will emerge at the classical level simply from a change of variables in a combination with a geometric interpretation, while the modulus of the Higgs field acquires a ground state expectation value from a non-Abelian generalization of (25).

We start by generalizing the construction of the gauge invariant supercurrent (18) to the case of the Weinberg-Salam model. We follow largely the approach in [18], with some minor changes that are convenient when we proceed to generalize the results of Section 2.

We start by decomposing the Higgs field Φ as follows,

$$\Phi = \phi \, \mathcal{X} \quad \text{with} \quad \phi = \rho \, e^{i\theta} \quad \& \quad \mathcal{X} = \mathcal{U} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{31}$$

Here ϕ is a complex field, $\mathcal{X}$ a two-component complex isospinor with $|\mathcal{X}| = 1$, and we take $\mathcal{U}$ to be a 2×2 $SU(2)$ matrix. The $SU_L(2) \times U_Y(1)$ gauge transformation acts on Φ as follows,

$$\Phi \; \to \; e^{i\omega_Y} \Omega \, \Phi \quad \Rightarrow \quad \begin{cases} \phi \; \longrightarrow \; e^{i\omega_Y} \phi \\ \mathcal{X} \; \longrightarrow \; \Omega \mathcal{X} \end{cases} \tag{32}$$

where $\Omega \in SU_L(2)$ and $e^{i\omega_Y} \in U_Y(1)$. The decomposition (31) also introduces a new (internal) *compact* gauge group

$$U_{\text{int}}(1) : \quad \begin{array}{c} \phi \to e^{-i\omega_c} \phi \\ \mathcal{X} \to e^{i\omega_c} \mathcal{X} \end{array} \tag{33}$$

which leaves the field Φ intact. Note that the spinor $\mathcal{X} \equiv \mathcal{X}_1$ and its isospin conjugate

$$\mathcal{X}_2 = i\tau_2 \mathcal{X}^*$$

form an orthonormal basis $(i, j = 1, 2$ and $a, b = \uparrow, \downarrow)$,

$$\mathcal{X}_i^\dagger \cdot \mathcal{X}_j \; \equiv \; \sum_{a=\uparrow,\downarrow} \mathcal{X}_{ia}^* \mathcal{X}_{aj} \; = \; \delta_{ij}$$

$$\sum_{i=1,2} \mathcal{X}_{ia} \mathcal{X}_{ib}^\dagger \; = \; \delta_{ab}$$

When we introduce the conjugate Higgs field

$$\Phi_c = \phi \mathcal{X}_2$$

we find the $SU_L(2) \times U_Y(1)$ supercurrents $(J_\mu^\pm, J_\mu^3)$ and $\mathcal{Y}_\mu$ (with $J_\mu^\pm = J_\mu^1 \pm iJ_\mu^2$) by expanding the covariant derivative of the Higgs field in the spinor basis $(\mathcal{X}_1, \mathcal{X}_2)$ [18]

$$D_\mu \Phi = \left[\frac{1}{\rho} \partial_\mu \rho - \frac{i}{2} \left(q J_\mu^3 - q' \mathcal{Y}_\mu \right) \right] \Phi + i\frac{q}{2} J_\mu^+ \cdot \Phi_c \tag{34}$$

Explicitely,

$$J_\mu^+ = -\frac{2i}{q} \mathcal{X}_2^\dagger \left(\partial_\mu + \frac{iq}{2} \vec{A}_\mu \cdot \vec{\tau} \right) \mathcal{X}_1 \equiv \vec{A}_\mu \cdot \vec{e}_+ + \frac{i}{q} \vec{e}_3 \cdot \partial_\mu \vec{e}_+ \,, \tag{35}$$

$$J_\mu^3 = -\frac{2i}{q} \mathcal{X}_1^\dagger \left(\partial_\mu + \frac{iq}{2} \vec{A}_\mu \cdot \vec{\tau} \right) \mathcal{X}_1 \equiv \vec{A}_\mu \cdot \vec{e}_3 - \frac{i}{2q} \vec{e}_- \cdot \partial_\mu \vec{e}_+ \,, \tag{36}$$

and

$$\mathcal{Y}_\mu = \frac{i}{q'|\phi|^2} \left[\phi^\star \left(\partial_\mu - i\frac{q'}{2} Y_\mu \right) \phi - c.c. \right] \tag{37}$$

and $\vec{e}_i$ $(i = 1, 2, 3)$ are three mutually orthogonal unit vectors defined by

$$\vec{e}_3 = -\frac{\Phi^\dagger \vec{\tau} \Phi}{\Phi^\dagger \Phi} \equiv -\mathcal{X}_1^\dagger \vec{\tau} \mathcal{X}_1 \tag{38}$$

$$\vec{e}_+ = \vec{e}_1 + i\vec{e}_2 = \mathcal{X}_2^\dagger \vec{\tau} \mathcal{X}_1 \tag{39}$$

The $SU(2)$ matrix $\mathcal{U}$ in (31) combines these into

$$\vec{e}_i \tau^i = \mathcal{U}^{-1} \vec{\tau} \mathcal{U} \tag{40}$$

and the 3×3 matrix $e_i{}^a$ is an element of $SO(3)$ since

$$e_i{}^a e_j{}^a = \delta_{ij} \qquad \& \qquad e_i{}^a e_i{}^b = \delta^{ab}$$

We view (35)-(39) as the following change of variables,

$$(\vec{A}_\mu, Y_\mu, \Phi) \to (J_\mu^3, J_\mu^\pm, \mathcal{Y}_\mu, \vec{e}_i, \rho) \tag{41}$$

On both sides of (41) there are sixteen real fields, and (41) is an invertible change of variables whenever $\rho \neq 0$; the Jacobian is ρ^3. When we substitute (41) in (27) we get

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[18]

$$\mathcal{L}_{\mathrm{WS}} = (\partial_\mu \rho)^2 + \lambda \rho^4 \; + \; \frac{1}{4}\left(\vec{G}_{\mu\nu}(\vec{J}) + \frac{4\pi}{q}\vec{\tilde{\Sigma}}_{\mu\nu}\right)^2 + \frac{1}{4}\left(F_{\mu\nu}(\mathcal{Y}) + \frac{4\pi}{q'}\tilde{\sigma}^\phi_{\mu\nu}\right)^2$$

$$+\frac{\rho^2}{4}\left(qJ^3_\mu - q'\mathcal{Y}_\mu\right)^2 + \frac{\rho^2 q^2}{4}J^+_\mu J^-_\mu \qquad (42)$$

Here $\vec{G}_{\mu\nu}$ and $F_{\mu\nu}$ are the field strength tensors of $\vec{J}_\mu$ resp. $\mathcal{Y}_\mu$,

$$\vec{G}_{\mu\nu}(\vec{J}) = \partial_\mu \vec{J}_\nu - \partial_\nu \vec{J}_\mu - q\,\vec{J}_\mu \times \vec{J}_\nu\,, \qquad (43)$$

$$F_{\mu\nu}(\mathcal{Y}) = \partial_\mu \mathcal{Y}_\nu - \partial_\nu \mathcal{Y}_\mu\,. \qquad (44)$$

The $\tilde{\sigma}^\phi_{\mu\nu}$ is the dual of the string tensor (21) in the present case and the $\vec{e}_i$ appear only through the singular quantity

$$\Sigma^i_{\mu\nu} = \frac{1}{8\pi}\epsilon^{ijk}(\vec{e}^j \cdot [\partial_\mu, \partial_\nu]\vec{e}^k) \qquad (45)$$

which is a non-Abelian generalization of (21).

We make the following two remarks:

1) If we resolve the relations (35), (36) for A^i_μ we can combine (35), (36) into

$$J^a_\mu = A^i_\mu e_i{}^a + \frac{1}{2q}\epsilon^{abc}e^j{}_b\partial_\mu e_{jc}$$

and when we invert this by using the fact that $e^i{}_a \in SO(3)$ we get

$$A^i_\mu = e^i{}_a J^a_\mu + e^i{}_a\frac{1}{2q}\epsilon^{abc}e^j{}_b\partial_\mu e_{jc} = e^i{}_a\{J^a_\mu + \frac{1}{2q}\epsilon^{abc}e^j{}_b\partial_\mu e_{jc}\} \qquad (46)$$

Here the second term is a pure gauge *i.e.* left-invariant Maurer-Cartan form,

$$(\epsilon^{abc}e^j{}_b\partial_\mu e_{jc}) \cdot e^i{}_a\frac{\tau^i}{2i} = \mathcal{U}^{-1}\partial_\mu\mathcal{U} \qquad (47)$$

where $\mathcal{U} \in SU(2)$ is defined in (31), (40).

2) Following (25) and [10]-[12] and [20] we propose that in the quantum theory the expectation values

$$< (qJ^3_\mu - q'\mathcal{Y}_\mu)^2 > = \pm\Delta^2_3 \qquad (48)$$

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$$< q^2 J_\mu^+ J_\mu^- > = \pm \Delta_\pm^2 \tag{49}$$

are non-vanishing, with the sign (in Minkowski space) depending on whether the condensate is space-like or time-like. From (42) we then estimate for the ground state value ρ_0 of the Higgs modulus

$$\rho_0^2 = \frac{1}{4\lambda}(\pm\Delta_3^2 \pm \Delta_\pm^2)$$

As in the Abelian case we again conclude that even though there is no bare Higgs mass, a non-vanishing Higgs condensate can be generated by the condensation of the intermediate vector bosons. Furthermore, the sign of the condensate *i.e.* whether we are in the broken or symmetric Higgs phase depends on the signs of the condensates (48), (49) that is whether we have a time-like or space-like condensate in the Minkowski space [20].

In line with (22) we can interpret the Lagrangian (27) in terms of local conformal geometry. As in (17) we identify ρ with the conformal scale of a metric tensor, and repeating the steps that led to (22) we get [18]

$$\mathcal{L}_{\rm WS} = \sqrt{G}\left\{\frac{1}{16\pi G}(R + 2\Lambda) + \mathcal{L}_M\right\} \tag{50}$$

where the matter Lagrangian $\mathcal{L}_M$ is

$$\mathcal{L}_M = \frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \kappa^2(q^2 + q'^2)Z_\mu Z^\mu + \kappa^2 q^2 W_\mu^+ W^{\mu-} \tag{51}$$

and the indices are raised and lowered using the metric tensor (17). As in (23), (24) we have here introduced

$$G = \frac{3}{8\pi\kappa^2}$$

and

$$\Lambda = 3\kappa^2\lambda$$

and the $SU_L(2) \times U_Y(1)$ invariant W–bosons are $W_\mu^\pm = J_\mu^\pm$, while the Z–boson and photon A_μ are

$$Z_\mu = \cos\theta_W J_\mu^3 - \sin\theta_W \mathcal{Y}_\mu\,, \tag{52}$$

$$A_\mu = \sin\theta_W J_\mu^3 + \cos\theta_W \mathcal{Y}_\mu\,, \tag{53}$$

where θ_W is the Weinberg angle, it has the experimental the low momentum transfer value [13]

$$\sin^2 \theta_W = \frac{q'^2}{q^2 + q'^2} = 1 - \frac{M_W^2}{M_Z^2} \approx 0.2397 \pm 0.0014 \tag{54}$$

By recalling the (low energy) Thomson limit value

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

for the electric charge this gives for the $SU_L(2) \times U_Y(1)$ couplings the following numerical values

$$e = q \sin \theta_W = q' \cos \theta_W \quad \Rightarrow \quad q \approx 0.619 \ \& \ q' \approx 0.312 \tag{55}$$

The Lagrangian (50), (51) has the familiar form of the spontaneously broken Einstein-Weinberg-Salam Lagrangian. It describes the conventional electroweak interactions of the massive W and Z bosons in a conformally flat space-time, that becomes a flat $\mathbb{R}^4$ in the London limit where ρ is a constant. As in the Abelian case, we find it notable that now it is the dimensionfull parameter κ that gives rise to the vector masses, *not* the Higgs ground state expectation value as in conventional approach. In fact, in line with the Abelian case we immediately observe that the presence of a bare Higgs mass term would not allow us to write the Lagrangian in the generally covariant form. While at the present level of argumentation this generally covariant form could be viewed as accidental, in the rest of this paper we argue that it may also reflect dimensional censorship imposed by an underlying higher dimensional structure. If so, the addition of a bare Higgs mass would lead to an explicit breaking of general covariance. But if the quantization is performed in a manner that respects general covariance as a Ward-Takahashi like identity, a Higgs mass term can not appear and the electroweak hierarchy problem may have a simple resolution. Furthermore, the conventional Higgs field becomes metamorphosed into the local conformal scale, and as in the Abelian case in the quantum theory its modulus may acquire a non-vanishing expectation value from the condensation of the intermediate vector bosons, without violating the underlying general covariance.

IV. SQUASHED THREE-SPHERE

We now proceed to disclose how the symmetry structure of the Weinberg-Salam Lagrangian in its representation (50), (51) becomes embedded in the brane world, to reflect the potential presence of higher dimensions. Following Section II we start with a Kaluza-Klein setup which we build on $\mathbb{M}^4 \times \mathbb{S}^3$. Here $\mathbb{M}^4$ is the space-time four-manifold with metric components $g_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3$). Eventually we shall specify to a locally conformally flat space-time, to reproduce the result (50), (51). The internal $\mathbb{S}^3 \sim SU(2)$ is the gauge group manifold that we eventually squash. It turns out that the squashing parameter will allow us to relate the gauge couplings q and q' in the Weinberg-Salam model.

In this Section we present some useful relations for $SU(2) \simeq \mathbb{S}^3$, both with the standard metric and its squashed generalization. The results are largely familiar [7] but there are some new details. We describe the manifold $SU(2) \simeq \mathbb{S}^3$ in terms of the 2×2 matrix $\mathcal{U}$ that we introduced in (40). For concreteness we use the following explicit Euler angle parametrization

$$\mathcal{U} = -i \begin{pmatrix} \sin \frac{\theta}{2} e^{\frac{i}{2}\phi_+} & -\cos \frac{\theta}{2} e^{\frac{i}{2}\phi_-} \\ -\cos \frac{\theta}{2} e^{-\frac{i}{2}\phi_-} & -\sin \frac{\theta}{2} e^{-\frac{i}{2}\phi_+} \end{pmatrix} \tag{56}$$

where $0 \leq \theta \leq \pi$ and $0 \leq \phi_\pm \leq 2\pi$ are local coordinates on $\mathbb{S}^3$. The natural metric g_{mn} ($m, n = 1, 2, 3$) on $\mathbb{S}^3$ is the bi-invariant Killing two-form,

$$ds^2 = 2\,Tr(d\mathcal{U}d\mathcal{U}^{-1}) = g_{mn}d\vartheta^m d\vartheta^n = (d\theta)^2 + \sin^2 \frac{\theta}{2}\,(d\phi_+)^2 + \cos^2 \frac{\theta}{2}\,(d\phi_-)^2 \tag{57}$$

We write the left-invariant Maurer-Cartan one-form (47) as follows,

$$\mathcal{U}^{-1}d\mathcal{U} = L^a_m d\vartheta^m \frac{1}{2i}\tau^a \tag{58}$$

where τ^a are the Pauli matrices. The right-invariant Maurer-Cartan is

$$\mathcal{U}d\mathcal{U}^{-1} = R^a_m d\vartheta^m \frac{1}{2i}\tau^a \tag{59}$$

The components L_m^a and R_m^a can both be identified as the dreibeins for the metric (57),

$$g_{mn} = \delta_{ab} L_m^a L_n^b = \delta_{ab} R_m^a R_n^b \qquad (60)$$

The one-forms $L^a = L_m^a d\vartheta^m$ and $R^a = R_m^a d\vartheta^m$ are also subject to the $SU_L(2)$ Maurer-Cartan equation, $e.g.$

$$dL^a = -\frac{1}{2}\epsilon^{abc} L^b \wedge L^c \qquad (61)$$

and explicitely we have

$$L^1 = e_3{}^1 d\psi_+ - e_2{}^1 d\theta \qquad (62)$$

$$L^2 = e_3{}^2 d\psi_+ - e_2{}^2 d\theta \qquad (63)$$

$$L^3 = e_3{}^3 d\psi_+ - d\psi_- \qquad (64)$$

where we have defined

$$\psi_\pm = \frac{1}{2}(\phi_+ \pm \phi_-)$$

and we have introduced the right handed orthonormal triplet (40),

$$\vec{e}_1 = \begin{pmatrix} \cos\psi_- \cos\theta \\ \sin\psi_- \cos\theta \\ -\sin\theta \end{pmatrix} \quad \& \quad \vec{e}_2 = \begin{pmatrix} -\sin\psi_- \\ \cos\psi_- \\ 0 \end{pmatrix} \quad \& \quad \vec{e}_3 = \begin{pmatrix} \cos\psi_- \sin\theta \\ \sin\psi_- \sin\theta \\ \cos\theta \end{pmatrix} \quad (65)$$

The ensuing explicit realizations of the right Maurer-Cartan one-forms are obtained simply by sending

$$(\theta, \psi_+, \psi_-) \to -(\theta, \psi_-, \psi_+) \qquad (66)$$

There are three left-invariant Killing vector fields

$$K_L^a = (K_L^a)^m \frac{\partial}{\partial \vartheta^m} \qquad (m = 1, 2, 3)$$

that can be identified as the canonical duals of the one-forms L^a. With (62)-(64) this gives us the explicit realization

$$K_L^1 = \left\{ \sin\psi_- \partial_\theta + \cos\psi_- \cot\theta \partial_{\psi_-} \right\} + \frac{\cos\psi_-}{\sin\theta}\partial_{\psi_+} = l^1 + t^1 \qquad (67)$$

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$$K_L^2 = \left\{ -\cos\psi_-\partial_\theta + \sin\psi_- \cot\theta\partial_{\psi_-} \right\} + \frac{\sin\psi_-}{\sin\theta}\partial_{\psi_+} = l^2 + t^2 \tag{68}$$

$$K_L^3 = -\partial_{\psi_-} \equiv l^3 \tag{69}$$

The commutators of the Killing vectors determine a representation of the $SU_L(2)$ Lie algebra,

$$[K_L^a, K_L^b] = -\epsilon^{abc}K_L^c \tag{70}$$

Furhermore, in l^a we identify the standard $SO(3)$ angular momentum operators with

$$[l^a, l^b] = -\epsilon^{abc}l^c$$

while the t^a obey the one-cocycle condition

$$[l^a, t^b] + [t^a, l^b] = -\epsilon^{abc}t^c \tag{71}$$

$$[t^a, t^b] = 0 \tag{72}$$

We also note the possibility to introduce a two-cocycle into the Lie algebra (70). For this we deform the Killing vectors into

$$K_L^a \to \hat{K}_L^a \equiv K_L^a + \alpha \cdot T_L^a = K_L^a + \alpha \cdot e_3^a \, \partial_{\psi_+} \tag{73}$$

The deformed Lie algebra is

$$[\hat{K}_L^a, \hat{K}_L^b] = -\epsilon^{abc}\hat{K}_L^c + \alpha \cdot \epsilon^{abc}T_L^c$$

and in the equivariant subspace where

$$T_L^a F(\theta, \psi_-, \psi_+) = 0 \Rightarrow F = F(\theta, \psi_-)$$

these deformed Killing vectors act like the original ones.

We recall that the Killing vectors generate an isometry of the metric (57). With $\mathcal{L}_a$ the Lie derivative in the direction of K_L^a

$$\mathcal{L}_a g_{mn} = 0 \qquad i = 1, 2, 3 \tag{74}$$

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They are also orthonormal,

$$g_{mn}(K_L^a)^m(K_L^b)^n = \delta^{ab} \tag{75}$$

Again, the ensuing explicit realization of the right Killing vectors is obtained from (66). In particular, from (69) we get

$$K_R^3 \equiv R = +\partial_{\psi_+} \tag{76}$$

We can explicitely break the $SU_L(2) \times SU_R(2)$ isometry of $\mathbb{S}^3$ into $SU_L(2) \times U_R(1)$ by squashing [7] the three sphere. For this we modify the metric tensor (57) into the following one-parameter family of metrics,

$$g_{mn} d\vartheta^m d\vartheta^n = (d\theta)^2 + \sin^2\theta \, (d\psi_-)^2 + (d\psi_+ - \cos\theta \, d\psi_-)^2 \tag{77}$$

$$\rightarrow (d\theta)^2 + \sin^2\theta \, (d\psi_-)^2 + \xi^2(d\psi_+ - \cos\theta \, d\psi_-)^2 = g_{mn}^\xi d\vartheta^m d\vartheta^n \tag{78}$$

A dreibein representation of this squashed metric is obtained *e.g.* in terms of the right Maurer-Cartan one-forms by modifying them as follows

$$E^1 = R^1 = e_3^1 \, d\psi_- - e_2^1 d\theta \tag{79}$$

$$E^2 = R^2 = e_3^2 \, d\psi_- - e_2^2 d\theta \tag{80}$$

$$E^3 = \xi \cdot R^3 = \xi \cdot (e_3^3 d\psi_- - d\psi_+) \tag{81}$$

where we have implemented the left-right conjugation (66) in the triplet (65). This gives the dreibein decomposition of the squashed metric tensor (78),

$$g_{mn}^\xi = E_m^i E_n^j \delta_{ij} = R_m^1 R_n^1 + R_m^2 R_n^2 + \xi^2 R_m^3 R_n^3 \tag{82}$$

Alternatively, we can introduce the following dreibein one-forms to similarly decompose the squashed metric,

$$E^1 = \left\{ e_3{}^1 \cos\theta d\psi_- - e_2{}^1 d\theta \right\} + \xi \, e_3{}^1 (d\psi_+ - \cos\theta d\psi_-) \tag{83}$$

$$E^2 = \left\{ e_3{}^2 \cos\theta d\psi_- - e_2{}^2 d\theta \right\} + \xi \, e_3{}^2 (d\psi_+ - \cos\theta d\psi_-) \tag{84}$$

$$E^3 = \left\{ e_3{}^3 \cos\theta d\psi_- - d\psi_- \right\} + \xi \, e_3{}^3 (d\psi_+ - \cos\theta d\psi_-) \tag{85}$$

These dreibeins have the advantage that in the $\xi \to 0$ limit none of them vanishes and they go smoothly over to give the standard metric on the two-sphere $\mathbb{S}^2$ with local coordinates (θ, ψ_-). This will become convenient in Section VIII.

Finally, we remind that for *any* value of the squashing parameter ξ in (78) the *original* left Killing vectors (67)-(69) in addition of the 3rd component of the right Killing vector (76) remain as the Killing vectors of the squashed sphere, independently of ξ. Together they generate the Lie algebra $SU_L(2) \times U_R(1)$. But since the $\psi_- \leftrightarrow \psi_+$ symmetry becomes broken for $\xi \neq 1$, the squashed three-sphere does not anymore admit the full right invariant $SU_R(2)$ isometry.

V. WEINBERG-SALAM AND SQUASHED SPHERE

We now generalize the derivation of (16) to inspect how the symmetry structure of Weinberg-Salam Lagrangian (50), (51) becomes embedded in the brane world. Our starting point is the *pure* seven dimensional Hilbert-Einstein action *without* a cosmological constant on the manifold $\mathbb{M}^4 \times \mathbb{S}^3_\xi$

$$S = \frac{1}{16\pi G} \frac{1}{V_\xi} \int d^4 x d^3 \vartheta \sqrt{g_{(7)}} \, R_{(7)} \tag{86}$$

We choose $\mathbb{M}^4$ to be a generic four-manifold with metric tensor $g_{\mu\nu}$ and local coordinates x^μ, and $\mathbb{S}^3_\xi$ is the squashed three-sphere now with metric

$$ds^2 = \frac{r^2}{4} g^\xi_{mn} d\vartheta^m d\vartheta^n = \frac{r^2}{4} \{ (d\theta)^2 + \sin^2\theta \, (d\psi_-)^2 + \xi^2 (d\psi_+ - \cos\theta \, d\psi_-)^2 \} \tag{87}$$

We take r to be a constant so that the volume of the squashed sphere is

$$V_\xi = 2\pi^2 \xi r^3$$

We introduce the following Kaluza-Klein decomposed metric over $\mathbb{M}^4 \times \mathbb{S}^3_\xi$

$$ds^2 = g_{\alpha\beta} dy^\alpha dy^\beta$$

$$= g_{\mu\nu} dx^\mu dx^\nu + \frac{r^2}{4} g^\xi_{mn} \{ d\vartheta^m + K_L^{am} A^a_\mu dx^\mu + R^m B_\mu dx^\mu \} \{ d\vartheta^n + K_L^{bn} A^b_\nu dx^\nu + R^n B_\nu dx^\nu \} \tag{88}$$

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Here K_L^{am} are the components of the left Killing vectors (67)-(69) and $R^m \equiv K_R^{3m}$ are the components of (76).

At this point we note the following: The decomposition (88) is *not* the most general one of the metric tensor, in particular it does not include the higher dimensional dilaton fields [27], [28]. However, here the goal is *not* to deduce the Weinberg-Salam model from a higher dimensional gravity theory using the Kaluza-Klein approach, this remains a problem that still waits for an elegant solution. Instead, as explained in the introduction we search for a resolution to the Higgs mass hierarchy problem from symmetries of the Weinberg-Salam Lagrangian that have a higher dimensional interpretation. We only inquire what is the minimal amount of higher dimensional structure that we need to look into, in order to address and hopefully resolve the electroweak hierarchy problem in the limited context of the bosonic Weinberg-Salam Lagrangian. In particular, how can we argue that the generally covariant form (50), (51) is not just an accidental coincidence that does not need to survive quantization, but a reflection of dimensional censorship imposed by an inherent higher dimensional symmetry structure that may be at the root of solving the electroweak hierarchy problem.

It is natural to assume that the higher dimensional manifold has the (local) product structure of a four-dimensional space-time manifold $\mathbb{M}_4$ with some internal manifold. Furthermore, whatever the structure of the internal manifold it should somehow relate to the squashed three-sphere, since the commutators of its Killing vectors coincide with the Lie algebra of $SU(2) \times U(1)$ [7]. Consequently we do not think it is unreasonable to assume that *any* approach to electroweak interactions that involves a higher dimensional construct, somehow relates to the structure of a squashed three sphere as an internal manifold, and for this reason we here select it as our internal manifold.

Note that as in the Abelian case we discussed in Section II, in a complete and fully consistent Kaluza-Klein approach where the dilaton fields are included we would arrive at an extension of the Weinberg-Salam model with additional scalar fields that

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are due to the dilatons [27], [28]. This can be of importance *if* LHC experiments observe signatures of unexpected scalar fields.

When we consider a coordinate transformation that sends

$$\delta\vartheta^m = -K_L^{am}\varepsilon^a(x^\mu) - R^m\varepsilon(x^\mu) \tag{89}$$

where $\epsilon^a(x^\mu), \epsilon(x^\mu)$ are arbitrary functions on $\mathbb{M}^4$, in direct generalization of (3) we find that the metric (88) remains intact provided

$$
\begin{aligned}
\delta A_\mu^a &= \partial_\mu\varepsilon^a + \epsilon^{abc}A_\mu^b\varepsilon^c \\
\delta B_\mu &= \partial_\mu\varepsilon
\end{aligned} \tag{90}
$$

This is the $SU_L(2) \times U_R(1)$ gauge transformation law of the gauge fields (A_μ^a, B_μ).

In order to perform the projection to massless states we assume that the $\mathbb{S}_\xi^3$ metric components g_{mn}^ξ and the components (K_L^{am}, R^m) of the Killing vectors depend solely on the three internal coordinates ϑ^m with no x^μ dependence, while $g_{\mu\nu}$ and (A_μ^a, B_μ) all depend only on the four dimensional x_μ. We substitute the metric (88) in (86) and we integrate over $\mathbb{S}_\xi^3$ to get

$$S = \frac{1}{\hbar}\int d^4x\sqrt{g}\left[\frac{1}{r^2}\{R + R_{int}\} + \frac{1}{4}\frac{\xi^2+2}{3}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} + \frac{1}{4}\xi^2 F_{\mu\nu}^2\right] \tag{91}$$

Here $\hbar$ is an *a priori* arbitrary dimensionless number, obtained by combining the various overall factors into a single quantity (we may call it a "Planck's constant"). All the metric structure is determined by the four dimensional $g_{\mu\nu}$, and $\vec{G}_{\mu\nu}$ is the $SU(2)$ field strength of $\vec{A}_\mu$ and $F_{\mu\nu}$ is the $U(1)$ field strength of B_μ. The internal scalar curvature is

$$R_{int} = \frac{4 - \xi^2}{2r^2}$$

and it has the rôle of a cosmological constant.

With (91), we now wish to recover the Weinberg-Salam Lagrangian (50), (51). For this we introduce the locally conformally flat metric tensor with components (17) and substitute in (91). The result is

$$S = \frac{1}{\hbar}\int d^4x\left\{\left[\frac{6}{\kappa^2}\frac{1}{r^2}(\partial_\mu\rho)^2 + \frac{\rho^4}{2r^4\kappa^4}(4-\xi^2)\right] + \frac{1}{4}\frac{\xi^2+2}{3}(\vec{G}_{\mu\nu})^2 + \frac{1}{4}\xi^2 F_{\mu\nu}^2\right\}$$

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This reproduces the first four terms in (42) (up to the overall dimensionless factor $\mathfrak{h}$) when we choose the (constant) radius r^2 to be

$$r^2 = \frac{6}{\kappa^2} \tag{92}$$

and we scale the gauge fields as follows,

$$\begin{aligned} A_\mu^a &\to q A_\mu^a \\ B_\mu &\to q' B_\mu \end{aligned} \tag{93}$$

where we select

$$q = \sqrt{\frac{3}{\xi^2 + 2}} \tag{94}$$

$$q' = \frac{1}{\xi} \tag{95}$$

and

$$\lambda = \frac{1}{4!} \frac{4 - \xi^2}{3} \tag{96}$$

In particular, these definitions ensure that the Yang-Mills contribution to the action acquires the correct canonical normalization (51),

$$S_{YM} = \int d^4 x \sqrt{g} \left\{ \frac{1}{4} \vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\} \tag{97}$$

Moreover, we note that the $SU_L(2)$ coupling q, the $U_R(1)$ coupling q' and the Higgs coupling λ are now *all* determined by the dimensionless squashing parameter ξ.

VI.   VECTOR BOSON MASS AND NAMBU BRANE

We proceed to construct the gauge invariant mass terms for the intermediate vector bosons. Following Section II we shall here show how a mass term can be obtained from a three-brane with Nambu action. From a geometrical point of view the Nambu action is a very natural choice. However, we shall find that it does not conform with the experimentally observed $W_\mu^\pm$ and Z_μ masses. The reason is that the Nambu action breaks an underlying local $SU_L(2) \times SU_R(2)$ custodial symmetry of the mass matrix.

In the next Section we show how the custodial symmetry is recovered and the correct intermediate vector boson masses obtained.

As in Section II we introduce a three-brane that stretches along the non-compact directions of $\mathbb{M}^4 \times \mathbb{S}^3_\xi$. Locally the brane is described by

$$\vartheta^m = X^m(x_\mu) \tag{98}$$

In analogy with (7) we introduce the basis vectors on the brane,

$$E_\mu{}^m = \delta_\mu{}^m + \frac{\partial X^m}{\partial x^\mu}$$

Together with (88) this leads to the induced brane metric

$$G^{ind}_{\mu\nu} = E_\mu{}^\alpha E_\nu{}^\beta g_{\alpha\beta} = g_{\mu\nu} + \frac{r^2}{4} g^\xi_{mn}(K_L^{am} A_\mu^a + R^m B_\mu + \partial_\mu X^m)(K_L^{bn} A_\nu^b + R^n B_\nu + \partial_\nu X^n) \tag{99}$$

in direct generalization of (8). We compute its determinant and the result is

$$\det[G^{ind}_{\mu\nu}] =$$

$$\det[g_{\mu\nu}] \cdot \left(1 + \frac{r^2}{4} g^{\mu\nu} g^\xi_{mn}(K_L^{am} A_\mu^a + R^m B_\mu + \partial_\mu X^m)(K_L^{bn} A_\nu^b + R^n B_\nu + \partial_\nu X^n)\right) \tag{100}$$

Here the three composites

$$\mathcal{J}_\mu^m = K_L^{am} A_\mu^a + R^m B_\mu + \frac{\partial X^m}{\partial x^\mu} \tag{101}$$

are the brane versions of the gauge invariant supercurrents (35), (36). By comparing (100) with (88) we conclude that these supercurrents are indeed invariant under the reparametrizations (89), (90) *a.k.a.* $SU_L(2) \times U_R(1)$ gauge transformations (recall that together (K_L^{am}, R^m) generate the unbroken $SU_L(2) \times U_R(1)$ isometry of $\mathbb{S}^3_\xi$). For example, in order to explicitly verify the invariance under the non-Abelian reparametrization (90) we first observe that

$$\delta(K_L^{am} A_\mu^a) = (\mathcal{L}_{-\varepsilon^b K_L^b} K_L^{am}) A_\mu^a + K_L^{am}(\partial_\mu \varepsilon^a + \epsilon^{abc} A_\mu^b \varepsilon^c)$$

$$= \epsilon^{abc} A_\mu^a \varepsilon^b K_L^{cm} + K_L^{am} \partial_\mu \varepsilon^a + \epsilon^{abc} \varepsilon^a A_\mu^b K_L^{cm} = K_L^{am} \partial_\mu \varepsilon^a$$

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On the other hand, from (98) we get by (89) that

$$\delta \left(\frac{\partial X^m}{\partial x_\mu} \right) = -K_L{}^{am} \partial_\mu \varepsilon^a$$

Furthermore, in line with (47) the last term in (101) is a pure gauge contribution. For this we recall (58) and (65), (76) to find

$$\mathcal{J}_\mu^m L_m^a \frac{\tau^a}{2i} = (A_\mu^a + B_\mu \, e_3{}^a) \frac{\tau^a}{2i} + \mathcal{U}^{-1} \partial_\mu \mathcal{U}$$

In the limit of small brane fluctuations the Nambu action for the brane can be expanded in derivatives of fluctuations and to leading nontrivial order we get

$$S_{brane} = \frac{1}{\hbar} T \int d^4 x \sqrt{G^{ind}} \approx \frac{1}{\hbar} T \int d^4 x \sqrt{g} \cdot \left(1 + \frac{1}{2} \frac{r^2}{4} g^{\mu\nu} g_{mn}^\xi \mathcal{J}_\mu^m \mathcal{J}_\nu^n + ... \right) \quad (102)$$

Here the first term contributes to the four dimensional cosmological constant and the second is the mass term for the supercurrents. We use (60) to write the mass term in (102) as follows,

$$\frac{T}{8} r^2 g^{\mu\nu} g_{mn}^\xi \mathcal{J}_\mu^m \mathcal{J}_\nu^n = g^{\mu\nu} \left(\frac{T}{8} r^2 E_m^i \delta_{ij} E_n^j \right) \mathcal{J}_\mu^m \mathcal{J}_\nu^n = g^{\mu\nu} M_{mn}(\xi) \mathcal{J}_\mu^m \mathcal{J}_\nu^n \quad (103)$$

where the E_m^i are the squashed dreibeins (79)-(81).

Since the mass term involves only three supercurrents, one linear combination of the four gauge fields A_μ^a, B_μ remains massless. To identify the massive and massless combinations we recall that the Kaluza-Klein reparameterizations *a.k.a.* gauge transformations act transitively and consequently we can (locally) introduce a coordinate transformation that makes the brane coordinates constants:

$$\theta = \psi_+ = \psi_- = 0$$

This amounts to rotating

$$\vec{e}_1 \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \& \quad \vec{e}_2 \rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \& \quad \vec{e}_3 \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

in (65); From the point of view of the original Weinberg-Salam model this corresponds to selecting the Unitary Gauge that always exists locally. We use (60) with the explicit realizations (83)-(85) and the rescaled fields (93) and diagonalize the mass matrix $M_{mn}(\xi)$ to conclude that the massless combination is

$$\mathcal{A}_\mu = \frac{qB_\mu - q'A_\mu^3}{\sqrt{q^2 + q'^2}} = -\sin\theta_W \cdot A_\mu^3 + \cos\theta_W \cdot B_\mu \tag{104}$$

and the massive combinations are

$$W_\mu^+ = A_\mu^1 + iA_\mu^2 \tag{105}$$

$$Z_\mu = \frac{q'B_\mu + qA_\mu^3}{\sqrt{q^2 + q'^2}} = \cos\theta_W \cdot A_\mu + \sin\theta_W \cdot B_\mu \tag{106}$$

where

$$\sin^2\theta_W = \frac{\xi^2 + 2}{4\,\xi^2 + 2}$$

so that

$$\frac{1}{4} \le \sin^2\theta_W \le 1$$

and we get from (102) the mass term

$$S_{mass} = \frac{1}{\hbar}\frac{r^2 T}{8} \int d^4x\sqrt{g} \cdot \left\{ q^2 W_\mu^+ W^{\mu-} + \xi^2(q^2 + q'^2)Z_\mu Z^\mu \right\} \tag{107}$$

By combining this with (91), (97) we get for the entire action in terms of the rescaled, canonical fields

$$S = \frac{1}{\hbar} \int d^4x\sqrt{g} \left\{ \frac{\kappa^2}{6} \left[R + \left(\frac{6T}{\kappa^2} + \frac{2}{4!}(4 - \xi^2)\kappa^2 \right) \right] + \frac{1}{4}\vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right.$$

$$\left. + \frac{3T}{4\kappa^2} \left\{ q^2 W_\mu^+ W^{\mu-} + \xi^2(q^2 + q'^2)Z_\mu Z^\mu \right\} \right\} \tag{108}$$

When we select the locally conformally flat metric tensor (17) and choose the parameters as in (92)-(96) and

$$T = \frac{4}{3}\kappa^4$$

we get a Lagrangian which is very similar in form to the Weinberg-Salam Lagrangian (42), with the Higgs coupling

$$\lambda = \frac{1}{4!}\frac{100 - \xi^2}{3}$$

In particular, in addition of the overall $\mathfrak{h}$ there are now only *two* independent parameters, κ that determines the mass scale and ξ that determines the three couplings q, q' and λ. The apparent difference between (42) and (108) is in the mass relations, they have the same form only when $\xi = 1$. But in this case we obtain the experimentally quite distant value

$$\sin^2 \theta_W = \frac{1}{2} \qquad (109)$$

for the Weinberg angle. Since we do not understand how to reconcile these differences we propose that the Nambu action is not the one realized in Nature to provide masses for the intermediate vector bosons.

VII. CUSTODIAL SYMMETRY

The mass matrix (103) is obtained from the induced metric using the Nambu action, and as such it has a very natural geometric origin. For a *generic* value of ξ it also shares the local $SU_L(2) \times U_R(1)$ isometry of the squashed three-sphere. But when $\xi = 1$ so that the metric tensor coincides with the bi-invariant (77), the symmetry of the mass matrix (103) becomes extended to the local $SU_L(2) \times SU_R(2)$ invariance and it can be presented entirely in terms of the $\mathbb{S}^3$ Killing vectors as follows,

$$M_{mn} = \frac{T}{8} r^2 L_m^i \delta_{ij} L_n^j = \frac{T}{8} r^2 R_m^i \delta_{ij} R_n^j = \frac{T}{16} r^2 (L_m^i \delta_{ij} L_n^j + R_m^i \delta_{ij} R_n^j) \qquad (110)$$

We call this local $SU_L(2) \times SU_R(2)$ symmetry of the mass matrix (110) the *custodial symmetry*. An unbroken custodial symmetry implies the following familiar relation between the intermediate vector boson masses and the Weinberg angle,

$$\sin^2 \theta_W = \frac{q'^2}{q^2 + q'^2} = \frac{\xi^2 + 2}{4\xi^2 + 2} = 1 - \frac{M_W^2}{M_Z^2} \qquad (111)$$

We also note that the custodial symmetry can be used to justify *a posteriori* the relative normalization of the Killing vectors that we have introduced in (88).

Since the squashed metric tensor (82) can be represented in terms of the $\mathbb{S}^3$ Killing vectors independently of ξ we may as well adopt the point of view that since the

Killing vectors determine the metric tensor they are more "primitive" and the mass matrix (110) is the most natural one also in the case of a squashed three-sphere, *irrespectively* of the value of ξ.

The most general mass matrix that breaks the custodial symmetry explicitely while retaining the $SU_L(2) \times U_R(1)$ symmetry is

$$M_{mn}(\eta) \;=\; \frac{T}{8}r^2 \left(R_m^1 R_n^1 + R_m^2 R_n^2 + \eta^2 R_m^3 R_n^3 \right) \tag{112}$$

Here η is a new parameter which is independent of the squashing parameter ξ. For $\eta = 1$ we have the custodial symmetry that becomes explicitely broken into $SU_L(2) \times U_R(1)$ for $\eta \neq 1$. Using the mass matrix (112) we introduce the following (Polyakov-like) brane action

$$S_{brane} \;=\; \frac{1}{\hbar}T \int d^4x \sqrt{g}\, g^{\mu\nu} M_{mn}(\eta) \mathcal{J}_\mu^m \mathcal{J}_\nu^n$$

With this we find instead of (108)

$$S = \frac{1}{\hbar} \int d^4x \sqrt{g} \left\{ \frac{\kappa^2}{6} \left[R + \frac{2}{4!}(4 - \xi^2)\kappa^2) \right] + \frac{1}{4}\vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right.$$
$$\left. + \kappa^2 \left[q^2 W_\mu^+ W^{\mu-} + \eta^2(q^2 + q'^2)Z_\mu Z^\mu \right] \right\} \tag{113}$$

where we continue to exclude a bare cosmological constant. This Lagrangian gives us the mass relation [29]

$$\eta^2 \cos^2 \theta_W \;=\; \eta^2 \frac{q^2}{q^2 + q'^2} \;=\; \frac{M_W^2}{M_Z^2}$$

with the experimental value [14]

$$\eta^2 \;=\; 1.01023 \pm 0.00022$$

We recall [14] that in the Standard Model the difference to the custodial symmetry value $\eta = 1$ is due to bosonic loops.

We now proceed to inspect the (classical) value of the Weinberg angle (111). For this we shall approximate $\eta = 1$. From (94) and (111) we find in the Thomson limit the value

$$q \sin \theta_W \;=\; \sqrt{\frac{3}{4\xi^2 + 2}} \;=\; e \;\Rightarrow\; \xi \;=\; \frac{1}{2}\sqrt{\frac{3}{e^2} - 2} \approx 2.77$$

so that

$$\sin^2 \theta_W \approx 0.296$$

and from (94), (95) we get

$$q = 0.557 \quad \& \quad q' = 0.361$$

These numbers are surprisingly close to the experimental low momentum transfer values (54), (55) in particular when we take into account that the present estimations are purely classical and in particular we have not taken into account any interactions nor any fermionic effects.

Furthermore, in the absence of a bare seven-dimensional cosmological constant we get from (96) the numerical value

$$\lambda = \frac{1}{4!} \frac{4 - \xi^2}{3} = -0.0511$$

which is small, but negative; Adding a small but positive bare cosmological constant would make the effective Higgs coupling positive but here we prefer to avoid this. Instead, we note that in the pure scalar $\lambda \phi^4$ field theory the four dimensional triviality is well established for bare $\lambda < 0$ [30] and this suggests that quantum effects could also here drive $\xi \to 2$.

Suppose now that we are in a conformally flat and Euclidean-Lorentz *i.e.* SO(4) invariant classical ground state of (113). The vector fields must all then vanish and when we substitute (17) in (113) we obtain the following equation for the conformal scale of the metric tensor (17),

$$-\Box \left(\frac{\rho}{\kappa} \right) + \frac{\kappa^2}{3 \cdot 4!} (4 - \xi^2) \left(\frac{\rho}{\kappa} \right)^3 = 0$$

This is solved by

$$ds^2 = \left(\frac{\rho}{\kappa} \right)^2 \eta_{\mu\nu} dx^\mu dx^\nu = \frac{\eta_{\mu\nu} dx^\mu dx^\nu}{\left[1 + \frac{4 - \xi^2}{(4!)^2} \kappa^2 \cdot x^2 \right]^2} \tag{114}$$

This gives us either the de Sitter or anti de Sitter metric as the ground state, depending on whether $\xi < 2$ or $\xi > 2$. These could be viewed as two different phases of the

theory, and the tricritical value $\xi = 2$ yields a flat $\mathbb{R}^4$ and corresponds to a Weinberg angle value

$$\sin^2 \theta_W \;=\; \frac{1}{3}$$

We note that according to our model this means that the Grand Unified prediction for the Weinberg angle [4]

$$\sin^2 \theta_W \;=\; \frac{3}{8}$$

corresponds to a phase which is *different* from the observed world.

Finally, since the (anti) de Sitter manifold is homogeneous and has constant curvature, we obtain a reasonable ground state expectation value for ρ assuming that we are in the vicinity of $x = 0$ in (114). This yields the estimate

$$< \rho > \;\approx\; \kappa$$

and gives us the standard relation between the value of the Higgs condensate and the intermediate vector boson masses. We conclude by noting, that in the quantum theory there will be corrections to this expectation value due to the intermediate vector boson condensates (48), (49).

VIII.   THE LIMIT OF TWO-SPHERE

In this Section we briefly consider the limit $\xi \to 0$ in the metric (78). This is of interest, since the limit represents a submanifold of the squashed three sphere that is the smallest manifold that allows a realization of the $SU(2) \times U(1)$ Lie algebra in terms of Killing vectors. In this limit we obtain the standard metric of $\mathbb{S}^2 \in \mathbb{R}^3$

$$ds^2 = \frac{r^2}{4} g_{mn} d\vartheta^m d\vartheta^n = \frac{r^2}{4} \left\{ (d\theta)^2 + \sin^2 \theta (d\psi_-)^2 \right\}$$

The dreibein (83)-(85) becomes

$$L^1 \to e_3{}^1 \cos\theta d\psi_- - e_2{}^1 d\theta$$

$$L^2 \to e_3{}^2 \cos\theta d\psi_- - e_2{}^2 d\theta$$

$$L^3 \to e_3{}^3 \cos\theta d\psi_- - d\psi_-$$

These are the dual to the three dimensional angular momentum operators l^a in (67)-(69) with respect to the $\mathbb{S}^2$ metric,

$$L^a_m = g_{mn}l^{an}$$

We remind that the two sphere is the coadjoint orbit of $SU(2)$ and so it supports a representation of $SU(2)$ which is given by the $\mathbb{S}^2$ Killing vectors $a.k.a.$ angular momentum operators l^a.

When we send $\xi \to 0$ in the Lagrangian (108) and remove the ψ_+ dependence, again rotating $\vec{e}_3$ to point towards the north pole we get

$$S = \frac{1}{\mathfrak{h}} \int d^4x \sqrt{-g} \left\{ \frac{1}{r^2}[R + R_{int}] + \frac{1}{4}\vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} + \frac{24\pi GT}{8}W_\mu W^{\mu\star} \right\} \qquad (115)$$

We observe that only two components of the $SU(2)$ gauge field are massive. This is the result we expect to have when we break $SU(2)$ into $U(1)$ in an Non-Abelian Higgs model, with the Higgs field in the adjoint representation of $SU(2)$.

IX. CONCLUSIONS

We have addressed the electroweak hierarchy problem posed by the instability of the Higgs mass by inspecting whether an apparent general covariance but with a locally conformally flat metric tensor that is present in the Weinberg-Salam Lagrangian, could be somehow interpreted in terms of full general covariance in a higher dimensional gravity theory. We have argued that if one starts from a seven dimensional generally covariant action that has the same form as the Hilbert-Einstein action and projects on a subset of its Kaluza-Klein decomposed fields, one arrives at the functional form of the Weinberg-Salam Lagrangian with correct vector boson masses but with no bare Higgs mass, as the presence of the Higgs mass is forbidden by dimensional censorship as it breaks general covariance of the gravity theory. Moreover, if the quantization of the electroweak theory can be performed so that a Ward-Takahashi like identity that ensures the preservation of the higher dimensional general covariance,

the Higgs mass remains absent and that could provide a resolution to the electroweak hierarchy problem. Indeed, it has been argued that the absence of a bare Higgs mass could help to resolve the gauge hierarchy problem [31] and it has also been argued that despite of the absence of the bare mass the eventual Higgs expectation value does not need to be small but can acquire a realistic value [32]. We have here shown that at the classical level this could be due to the (anti) de Sitter ground state of the theory, while in the quantum theory a nontrivial expectation value for the Higgs condensate could emerge from the A^2 condensation of the intermediate vector bosons.

This line of arguments, if validated in the context of electroweak theory, would give us the first indication that four dimensional physics is sensitive to higher dimensional symmetry structure, even though it may remain questionable whether the Weinberg-Salam model with its fermions indeed emerges from a Kaluza-Klein reduction of some complete and internally consistent higher dimensional theory theory.

In addition, we have found that the higher dimensional general covariance enforces us a relation between the two electroweak gauge couplings, and the Higgs self-coupling. They are all determinants of a single parameter, the squashing parameter of the internal three sphere. Furthermore, when we use the known low energy value 1/137 of the electromagnetic fine structure constant our approach predicts the value $\sin^2 \theta_W = 0.296$ for the Weinberg angle. Potentially this value could be brought even closer to the observed value by inclusion of interactions, quantum effects and fermions, thus it may serve as an experimental test of the validity of our higher dimensional approach.

An interesting peculiarity in our approach is the absence of a primordial Higgs field. The modulus of Higgs field resides in the conformal scale of the four dimensional metric and as such it has no direct rôle in the mass generation of the vector fields. Instead the intermediate vector bosons acquire their masses from a three brane that asymptotically coincides with the physical space-time but is locally allowed to fluctuate into higher dimensions.

Except for the relation between the coupling constants that should eventually

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become an experimental test between our approach and the standard electroweak theory, the phenomenological content of the present Kaluza-Klein based electroweak theory appears to be very similar to that of the conventional Weinberg-Salam model. But we also note that there could be subtle differences [33] that might become visible at the LHC experiments. In particular, the potential observation of additional neutral scalar particles at LHC besides the modulus of the Higgs could have an interpretation either in terms of the two non-conformal modes that describe the physical field degrees of freedom of our four dimensional Hilbert-Einstein action (113) or in terms of the higher dimensional dilaton fields that are present if the higher dimensional generally covariant theory is interpreted literally in the conventional Kaluza-Klein sense.

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$$< T^\mu_{\ \mu} > \propto\ aTr\{F^2_{\mu\nu}\} + bR^2 + ...$$

The potential relation between Weyl anomaly and Higgs mass in the present context remains to be clarified.

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